Introduction to analysis of the infinite euler pdf

This is another large project that has now been completed : this completes in its entirety Euler's magnificent contribution to the establishment of teaching books on analysis and calculus. John D. Blanton has already translated Euler's Introduction to Analysis and approx. one third of Euler's monumental Foundations of Differential Calculus into English : I have decided not even to refer to these translations; any mistakes made can be corrected later. I hope that some people will come with me on this great journey : along the way, if you are unhappy with something which you think I have got wrong, please let me know and I will fix the problem a.s.a.p. There are of course, things that we now consider Euler got wrong, such as his rather casual use of infinite quantities to prove an argument; these are put in place here as Euler left them, perhaps with a note of the difficulty. It is not the business of the translator to 'modernize' old texts, but rather to produce them in close agreement with what the original author was saying. These works are available in the public domain on the Euler Archive ; a PDF file of the original Euler work in Latin, together with files of full translations into French (by J. B. Labey in 1796; 2 volumes) & German (by J. A. C. Michelsen in 1788–91, 3 volumes) are currently available to download for personal study at the e-rara.com website. Volumes I and II are now complete. The appendices will follow later. Volume I, Section I. Click here for Euler's Preface relating to volume one. Click here for the 1st Chapter : Concerning the kinds of functions. Euler starts by defining constants and variables, proceeds to simple functions, and then to multi–valued functions, with numerous examples thrown in. Click here for the 2nd Chapter : The transformation of functions. Most of this chapter is concerned with showing how to expand fractional functions into a finite series of simple terms, of great use in integration, of course, as he points out. Click here for the 3rd Chapter : The transformation of functions by substitution. This is a fairly straight forwards account of how to simplify certain functions by replacing a variable by another function of a new variable : essentially a function of a function ; in particular some carefully chosen series expansions in two variables can be simplified by a particular kind of substitution, enabling the original variables to be found explicitly in terms of the new one. Click here for the 4th Chapter : The development of functions into infinite series. This is also straight forwards ; simple fractional functions are developed into infinite series, initially based on geometric progressions. Comparisons are made with a general series and recurrent relations developed ; binomial expansions are introduced and more general series expansions presented. Click here for the 5th Chapter : Functions of two or more variables. Euler extends his classification to functions of two or more variables; some time is spent on homogeneous functions; occasional terms are introduced that are no longer used in this classification. Click here for the 6th Chapter : Concerning exponential and logarithmic functions The exponential and logarithmic functions are introduced, as well as the construction of logarithms from repeated square root extraction. Some problems are solved involving population growth and compound interest calculations. Click here for the 7th Chapter : Establishing logarithmic and exponential functions in series The familiar exponential function is finally established as an infinite series, as well as the series expansions for natural logarithms. Katz's A History of Mathematics, Ch. 12, p. 513 sets out some of the results from this chapter, which have been carried forwards to the present day without much change. The relation between natural logarithms and those to other bases are investigated, and the ease of calculation of the former is shown. Click here for the 8th Chapter : On transcending quantities arising from the circle. The ideas presented in the preceding chapter flow on to measurements of circular arcs, and the familiar expansions for the sine and cosine, tangent and cotangent, etc. Another brilliant chapter. Click here for the 9th Chapter : Concerning the investigation of trinomial factors. Initially polynomials are investigated to be factorized by linear and quadratic terms, using complex algebra to find the general form of the latter. The analysis is continued into infinite series using the familiar limiting form of the exponential function, to give series and infinite products for the modern hyperbolic sinh and cosh functions, as well as the sine and cosine. Click here for the 10th Chapter : Concerning the use of the factors found above in defining the sums of infinite series. Use is made of the results in the previous chapter to evaluate the sums of inverse powers of natural numbers; numerous well–known formulas are to be found here. Euler goes as high as the inverse 26th power in his summation. Click here for the 11th Chapter : Concerning other infinite products of arcs and sines. This is a most interesting chapter, as in it Euler shows the way in which the logarithms, both hyperbolic and common, of sines, cosines, tangents, etc. may be constructed into a canon of sines, cosines, etc., together with their logarithms, using infinite sums of reciprocal integers found by Euler. Click here for the 12th Chapter : Concerning the expansion of fractional functions. In this chapter Euler exploits his mastery of complex forms to elaborate on a procedure for extracting finite expansions from whole or algebraic functions, to produce finite series with simple or quadratic denominators; all of which of course have a bearing on making such functions integrable. Click here for the 13th Chapter : About recurring series. In this chapter sets out to show how the general terms of recurring series, developed from a simple division of numerator by denominator, can be found alternatively from expansions of the terms of the denominator, factorized into simple and quadratic terms, and by comparing the coefficient of the nth from the direct division with that found from this summation process, which in turn has been set out in previous chapters. Click here for the 14th Chapter : The multiplication and division of angles. In this chapter, which is a joy to read, Euler sets about the task of finding sums and products of multiple sines, cosines, tangents, etc. This is vintage Euler, doing what he was best at, presenting endless formulae in an almost effortless manner! The sums and products of sines to the various powers are related via their algebraic coefficients to the roots of associated polynomials. This chapter still has meaning for the math. student. Click here for the 15th Chapter : Series arising from the expansion of factors. In this chapter, Euler expands inverted products of factors into infinite series and vice versa for sums into products; he dwells on numerous infinite products and series involving reciprocals of primes, of natural numbers, and of various subsets of these, with plus and minus signs attached. This is an endless topic in itself, and clearly was a source of great fascination for him; and so it was for those who followed. Click here for the 16th Chapter : Concerning the partition of numbers. In this chapter, Euler develops the generating functions necessary, from very simple infinite products, to find the number of ways in which the natural numbers can be partitioned, both by smaller different natural numbers, and by smaller natural numbers that are allowed to repeat. To this are added some extra ways of subdividing. Click here for the 17th Chapter : The use of recurring series in investigating the roots of equations. In this chapter, Euler develops an idea of Daniel Bernoulli for finding the roots of equations. The reciprocal of a polynomial, for example, is expressed as a product of the roots, initially these are assumed real and simple, and which are then expanded in infinite series. The largest root can be found from the ratio of succeeding terms, etc. Troubles emerge when multiple roots occur, and/or complex roots, which are carefully analyzed to find the usefulness of the method. Click here for the 18th Chapter : Continued fractions. In this chapter, Euler develops the idea of continued fractions. This truly one of the greatest chapters of this book, and can be read with complete understanding by almost anyone. The concept of continued fractions is introduced and gradually expanded upon, so that one can change a series into a continued fraction, and vice-versa; quadratic equations can be solved, and decimal expansions of e and pi are made. This is the final chapter in Book I. Volume II, Section I. Click here for the 1st Chapter : About curved lines in general. Euler starts by setting up what has become the customary way of defining orthogonal axis and using a system of coordinates. He then applies some simple rules for finding the general shapes of continuous curves of even and odd orders in y. Click here for the 2nd Chapter : About changing coordinates. Euler shows how both orthogonal and skew coordinate systems may be changed, both by changing the origin and by rotation, for the same curve. This is done in a very neat manner. Click here for the 3rd Chapter : Concerning the division of algebraic curved lines into orders. The vexing question of assigning a unique classification system of curves into classes is undertaken here; with some of the pitfalls indicated; eventually a system emerges for algebraic curves in terms of implicit equations, the degree of which indicates the order; however, even this scheme is upset by factored quantities of lesser orders, representing the presence of curves of lesser orders and straight lines. Click here for the 4th Chapter : Concerning the particular properties of the lines of each order. This is a straight forwards chapter in which Euler examines the implicit equations of lines of various orders, starting from the first order with straight or right lines. Coordinate systems are set up either orthogonal or oblique angled, and linear equations can then be written down and solved for a curve of a given order passing through the prescribed number of given points. Click here for the 5th Chapter : Concerning lines of the second order. This is a rather mammoth chapter in which Euler examines the general properties of curves of the second order, as he eventually derives the simple formula for conic sections such as the ellipse; but this is not achieved without a great deal of argument, as the analysis starts from the simple basis of a line cutting a second order curve in two points, the sum and product of the lengths being known. A great deal of work is done on theorems relating to tangents and chords, which could be viewed as extensions of the more familiar circle theorems. However, the eccentricity definition of the conic, as the ratio of the distance of a point on the curve to the focus to the distance to the latus rectum, has not yet been found ; one might wonder how Euler came to miss this, considering his astonishing powers. Click here for the 6th Chapter : The subdivision of lines of the second order into kinds. This is a follow up chapter to the above, in which Euler examines the properties of particular curves that arise when the general equation of the second order in the plane is restricted in some way; this gives rise to the familiar conic sections, and Euler's is one of the best you will come across, without introducing the idea of eccentricity. Click here for the 7th Chapter : The investigation of infinite branches. This chapter examines the nature of curves of any order expressed by two variables, when such curves are extended to infinity. After some initial spade work, Euler shows how to express such asymptotic curves in terms of variables related to straight line asymptotes; a systematic method is then developed for dealing with curves up the fourth order, showing their asymptotic nature, but which could be extended easily to higher orders. Click here for the 8th Chapter : Concerning asymptote lines. This chapter essentially is an extension of the last above, where the business of establishing asymptotic curves and lines is undertaken in a most thorough manner, without of course referring explicitly to limiting values, or even differentiation; the work proceeds by examining changes of axes to suitable coordinates, from which various classes of straight and curved asymptotes can be developed. It is perhaps a good idea to start by looking at Euler's final single example, where he show how the various rules can be used in finding the asymptotes of a certain function of x and y , which he has produced for this very task. Click here for the 9th Chapter : The subdivision of lines of the third order into kinds. This chapter proceeds, after examining curves of the second order as regards asymptotes, to establish the kinds of asymptotes associated with the various kinds of curves of this order; essentially an application of the previous chapter. At the end, Euler compares his subdivision with that of Newton for curves of a similar nature. Click here for the 10th Chapter : The principal properties of lines of the third order. This chapter proceeds from the last, setting out the properties of such curves, and grouping them into different kinds; there is a tie up with Newton's classification of such curves, which I have not yet had time to investigate. Please write to me if you are knowing about such things, and wish to contribute something meaningful to this translation. Click here for the 11th Chapter : Lines of the fourth order. This chapter proceeds as the last; however, now the fundamental equation has many more terms, and there are over a hundred possible asymptotes of various forms, grouped into genera, within which there are kinds. In some respects this chapter fails, as it does not account for all the asymptotes, as the editor of the O.O. edition makes clear. However, it has seemed best to leave the exposition as Euler presented it, rather than to spent time adjusting the presentation, which one can find more modern texts. Click here for the 12th Chapter : Concerning the investigation of the figures of curved lines. This chapter proceeds from the previous one, and now the more difficult question of finding the detailed approximate shape of a curved line in a finite interval is considered, aided of course by the asymptotic behavior found above more readily. Euler produces some rather fascinating curves that can be analyzed with little more than a knowledge of quadratic equations, introducing en route the ideas of cusps, branch points, etc. A definite must do for a beginning student of mathematics, even today! Click here for the 13th Chapter : Towards an understanding of curved lines. This is an amazingly simple chapter, in which Euler is able to investigate the nature of curves of the various orders without referring explicitly to calculus; he does this by finding polynomials of appropriate degrees in (t, u) which are vanishingly small coordinates attached to the curve near an origin M, also on the curve. Click here for the 14th Chapter : The curvature of curved lines. This is a longer and thoughtful chapter, in which Euler investigates simple curves that 'fit' a given curve in a region around some point on the curve; he finds that the simplest such curve is a parabola, but demurs to consider the conventional circle of curvature, which is seen to be an artifice rather than a natural consequence. Any point on a curve can be one of three kinds : either a simple point, a point of inflection, or a cusp; with possible subdivisions. Click here for the 15th Chapter : Concerning curves with one or more given diameters. This is another long and thoughtful chapter, in which Euler investigates types of curves both with and without diameters; the coordinates chosen depend on the particular symmetry of the curve, considered algebraic and closed with a finite number of equal parts. At the end curves with cusps are considered in a similar manner. Click here for the 16th Chapter : Finding curves from the given properties of applied lines. This is another long and thoughtful chapter ; here Euler considers curves which are quadratic, cubic, and higher order polynomials in the variable y, and the coefficients of which are rational functions of the abscissa x; for a given x, the equation in y equated to zero gives two, three, or more intercepts for the y coordinate, or the applied line in 18th century speak. New curves are found by changing the symmetric functions corresponding to the coefficients of these polynomials, expressed as sums and products of these functions. This becomes progressively more elaborate as we go to higher orders; finally, the even and odd properties of functions are exploited to find new functions associated with two abscissas, leading in one example to a constant product of the applied lines, which are generalized in turn. Click here for the 17th Chapter : Finding curves from properties of applied lines. This is another long and thoughtful chapter ; this time a more elaborate scheme is formulated for finding curves; it involves drawing a line to cut the curve at one or more points from a given point outside or on the curve on the axis, each of which is detailed at length. This involves establishing equations of first, second, third, etc. orders or intersection points, the roots of which are the intercepted distances. This chapter is harder to understand at first because of the rather abstract approach adopted initially, but bear with it and all becomes light in the end. It is perhaps a good idea to look at the trisection of the line first, where the various conditions are set out, e.g. the sum of the lengths is constant, etc. Click here for the 18th Chapter : Concerning the similarity and affinity of curved lines. This is a much shorter and rather elementary chapter in some respects, in which the curves which are similar are described initially in terms of some ratio applied to both the x and y coordinates of the curve ; affine curves are then presented in which the ratios are different for the abscissas and for the applied lines or y ordinates. Finally, ways are established for filling an entire region with such curves, that are directed along certain lines according to some law. Click here for the 19th Chapter : The intersection of curves. In this chapter Euler investigates the possibilities of two curves intersecting; starting from humble beginnings with circles and parabolas, his analysis runs into difficulties associated with the presence of imaginary applied lines, double point, and so on, in his defining equations. Eventually he concentrates on a special class of curves where the powers of the applied lines y are increased by one more in the second uniform curve than in the first, and where the coefficients are functions of x only; by careful algebraic manipulation the powers of y can be eliminated while higher order equations in the other variable x emerge. To this theory, another more sophisticated approach is appended finally, giving the same results. Click here for the 20th Chapter : The construction of equations. In this chapter Euler investigates how equations can arise from the intersection of known curves, for which the roots may be known or found easily. On the one hand we have here the elements of the coordinate geometry of simple curves such as conic sections and curves of higher order, as well as ways of transforming equations into the intersection of known curves of higher orders, while attending to the problems associated with imaginary roots. Click here for the 21th Chapter : Concerning transcending curved lines. In this penultimate chapter Euler opens up his glory box of transcending curves to the mathematical public, and puts on show some of the splendid curves that arose in the early days of the calculus, as well as pointing a finger towards the later development of curves with unusual properties. Most impressive! Click here for the 22nd Chapter : The solution of some problems relating to the circle. In the final chapter of this work, numerical methods involving the use of logarithms are used to solve approximately some otherwise intractable problems involving the relations between arcs and straight lines, areas of segments and triangles, etc, associated with circles. The appendices to this work on surfaces I hope to do a little later. Volume II, Appendices on Surfaces. Click here for the 1st Appendix : About surfaces in general. It is amazing how much can be extracted from so little! In this first appendix space is divided up into 8 regions by a set of orthogonal planes with associated coordinates; the regions are then connected either by adjoining planes, lines, or a single point. Click here for the 2nd Appendix : The intersections of any surfaces made in general by some planes. Here the manner of describing the intersection of planes with some solid volumes is introduced with relevant equations. Click here for the 3rd Appendix : The intersections of the cylinder, cone, and sphere. Here the manner of describing the intersection of a plane with a cylinder, cone, and sphere is set out. This chapter contains a wealth of useful material; for the modern student it still has relevance as it shows how the equations of such intersections for the most general kinds of these solids may be established essentially by elementary means; it would be most useful, perhaps, to examine the last section first, as here the method is set out in general, before embarking on the rest of the chapter. The work on the scalene cone is perhaps the most detailed, leading to the various conic sections. Click here for the 4th Appendix : The changing of coordinates. This appendix looks in more detail at transforming the coordinates of a cross-section of a solid or of the figure traced out in a cross-section. Euler has produced the customary rotation/translation formulas for changing the axes in a plane, and also for changing from one plane to another in three dimensions, from elementary geometry. Click here for the 5th Appendix : Surfaces of the second order. This appendix follows on from the previous one, and is applied to second order surfaces, which includes the introduction of a number of the well-known shapes now so dear to geometers in this computing age. It is of interest to see how Euler handled these shapes, such as the different kinds of ellipsoid, paraboloid, and hyperboloid in three dimensional diagrams, together with their cross-sections and asymptotic cones, where appropriate. Click here for the 6th Appendix : The intersection of two surfaces. This appendix extends the above treatments to the examination of cases in three dimensions, including the intersection of curves in three dimensions that do not have a planar section. Thus Euler ends this work in mid-stream as it were, as in his other teaching texts, as there was no final end to his machinations ever..... This completes my present translations of Euler. 'Tomorrow, to fresh fields, and pastures new'. Ian Bruce. Jan. 16th , 2013 latest revision. Please feel free to contact me if you wish by clicking on my name here, especially if you have any relevant comments or concerns.